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Hamiltonian System Mechanics on (2,0)-Jet Bundles

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Abstract

The goal of this paper is to present Hamiltonian system Mechanics on (2,0)-jet bundles . In conclusion, some differential geometrical and physical results on the related mechanic systems have been given.

Keywords: Jet bundle; holomorphic bundle; complex , Hamiltonian Dynamics.

1. Introduction

It is well known that the dynamics of Lagrangian formalisms is characterized by a suitable vector field defined on the tangent and cotangent bundles which are phase-spaces of velocities and momentum of a given configuration manifold. If $\mathcal{M}$ is an m-dimensional configuration manifold [6]. If $\mathbf{H}: \mathbf{T}^*\mathcal{M} \longrightarrow \mathbf{R}$ is a regular Hamiltonian function then there is a unique vector field Z_H on cotangent bundle $\mathbf{T}^*\mathcal{M}$ such that dynamical equations

$$\mathbf{i}_{Z_H}\boldsymbol{\phi} = \mathbf{dH} \qquad (1)$$

where $\boldsymbol{\phi}$ is the symplectic form and H stands for Hamiltonian function. The paths of the Hamiltonian vector field Z_H are the solutions of the Hamiltonian equations shown by

$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \quad , \quad \frac{dp_i}{dt} = \frac{\partial H}{\partial q^i} \qquad (2)$$

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where q^i and $(q^i, p_i), 1 \leq i \leq m$, are coordinates of $\mathcal{M}$ and $\mathbf{T}^*\mathcal{M}$. The triple $(\mathbf{T}^*\mathcal{M}, \phi, H)$, is called Hamiltonian system on the cotangent bundle $\mathbf{T}^*\mathcal{M}$ with symplectic form ϕ. Let $\mathbf{T}^*\mathcal{M}$ be symplectic manifold with closed symplectic form ϕ. In this paper related to Hamiltonian equations Hamiltonian system Mechanics on (2,0)-jet bundles.

2. The geometry of holomorphic $J^{(2,0)}\mathcal{M}$ bundles

2.1 Definition

Let $\mathcal{M}$ be a complex manifold, $\mathbf{T_c}\mathcal{M} = \acute{\mathbf{T}}\mathcal{M} \oplus \acute{\acute{\mathbf{T}}}\mathcal{M}$, the complexified tangent bundle of $(\mathbf{1},\mathbf{0})$- and of $(\mathbf{0},\mathbf{1})$-type vectors, respectively. If $(\mathbf{z^i})_{\mathbf{i}=\overline{\mathbf{1;n}}}$ are complex coordinates, then $\acute{\mathbf{T}}_\mathbf{z}\,\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial \mathbf{z^i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ and $\acute{\acute{\mathbf{T}}}_\mathbf{z}\mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial \bar{\mathbf{z}}^\mathbf{i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ moreover $\acute{\mathbf{T}}\mathcal{M}$ is a holomorphic vector bundle

let $\mathbf{Z} = (\mathbf{z^i}, \mathbf{X^i} = \boldsymbol{\eta}^{\mathbf{i}(\mathbf{1})} = \frac{\mathbf{dz^i}}{\mathbf{d\theta}}, \mathbf{Y^i} = \boldsymbol{\eta}^{\mathbf{i}(\mathbf{2})} = \frac{\mathbf{d^2z^i}}{\mathbf{d\theta^2}})$ be local complex coordinates in the chart $(\mathbf{U};\ \boldsymbol{\psi})$ from $\mathbf{J}^{(\mathbf{2,0})}\mathcal{M}$;

we shall the following notations [1].

$$Z = \left(z^i, x^i = \eta^{i(1)}, y^i = \eta^{i(2)}\right) = (z^i, X^i, Y^i) \qquad (3)$$

2.2 Theorem

A local basis in $\acute{\mathbf{T}}_\mathbf{z}\left(\mathbf{J}^{(\mathbf{2,0})}\mathcal{M}\right)$ is $\left\{\frac{\partial}{\partial \mathbf{z^i}}, \frac{\partial}{\partial \mathbf{x^i}}, \frac{\partial}{\partial \mathbf{y^i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$ and in $\acute{\acute{\mathbf{T}}}_\mathbf{z}\left(\mathbf{J}^{(\mathbf{2,0})}\mathcal{M}\right)$ theirs conjugates $\left\{\frac{\partial}{\partial \bar{\mathbf{z}}^\mathbf{i}}, \frac{\partial}{\partial \bar{\mathbf{x}}^\mathbf{i}}, \frac{\partial}{\partial \bar{\mathbf{y}}^\mathbf{i}}\right\}_{\mathbf{i}=\overline{\mathbf{1;n}}}$: Due to holomorphic changes on $\mathbf{J}^{(\mathbf{2,0})}\mathcal{M}$, that is all of $\frac{\partial \acute{\mathbf{z}}^\mathbf{i}}{\partial \bar{\mathbf{z}}^\mathbf{j}}, \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \bar{\mathbf{z}}^\mathbf{j}}, \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \bar{\mathbf{z}}^\mathbf{j}}, \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \bar{\mathbf{x}}^\mathbf{j}}, \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \bar{\mathbf{x}}^\mathbf{j}}, \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \bar{\mathbf{y}}^\mathbf{j}}$ j are vanishing, and also theirs conjugates, it follows that local bases from $\acute{\mathbf{T}}_\mathbf{z}\left(\mathbf{J}^{(\mathbf{2,0})}\mathcal{M}\right)$ change w.r.t. the transformations by the rules:

$$\frac{\partial}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{z}}^\mathbf{i}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{z}}^\mathbf{i}} + \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{x}}^\mathbf{i}} + \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \mathbf{z^j}}\overset{\frac{\partial}{\partial \acute{\mathbf{y}}^\mathbf{i}}}{\Rightarrow}$$

$$\frac{\partial}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{x}}^\mathbf{i}} + \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{y}}^\mathbf{i}}$$

$$\frac{\partial}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \mathbf{z^j}}\frac{\partial}{\partial \acute{\mathbf{y}}^\mathbf{i}} \qquad (4)$$

Infer that $\frac{\partial \acute{\mathbf{z}}^\mathbf{i}}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \mathbf{x^j}} = \frac{\partial \acute{\mathbf{y}}^\mathbf{i}}{\partial \mathbf{y^j}}$ but in change $\frac{\partial \acute{\mathbf{z}}^\mathbf{i}}{\partial \mathbf{z^j}} = \frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \mathbf{x^j}}$ contain the second order derivatives of $\acute{\mathbf{z}}^\mathbf{i}$. while $\frac{\partial \acute{\mathbf{x}}^\mathbf{i}}{\partial \mathbf{z^j}}$ contains even the 3-th derivatives of $\acute{\mathbf{z}}^\mathbf{i}$.

2.3 Theorem

On $\boldsymbol{T_c}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$ the natural complex structure $\boldsymbol{J^2=-I}$ acts as follows:

$$J\left(\frac{\partial}{\partial z^j}\right)=\boldsymbol{i}\frac{\partial}{\partial z^j}\quad,\ J\left(\frac{\partial}{\partial x^j}\right)=\boldsymbol{i}\frac{\partial}{\partial x^j}\quad,\ J\left(\frac{\partial}{\partial y^j}\right)=\boldsymbol{i}\frac{\partial}{\partial y^j}$$

$$\boldsymbol{J\left(\frac{\partial}{\partial \bar{z}^j}\right)=-i\frac{\partial}{\partial z^j}\quad,\ J\left(\frac{\partial}{\partial \bar{x}^j}\right)=-i\frac{\partial}{\partial x^j}\quad,\ J\left(\frac{\partial}{\partial \bar{y}^j}\right)=-i\frac{\partial}{\partial y^j}}\quad(5)$$

The dual endomorphism the cotangent space $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$at any point p of manifold $\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}$ satifies $\mathbf{J^{2^*}=-I}$ and is defined by

$$\boldsymbol{J^*(dz^j)=idz^j\quad,\quad J^*(dx^j)=idx^j\quad,\ J^*(dy^j)=idy^j}$$

$$\boldsymbol{J^*(d\bar{z}^j)=-id\bar{z}^j\quad,\quad J^*(d\bar{x}^j)=-id\bar{x}^j\quad,\ J^*(\bar{y}^j)=-id\bar{y}^j}\quad(6)$$

3. Hamiltonian Dynamical Systems

In this section, we obtain complex Hamiltonian equations for classical mechanics structured on momentum space $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$ that is 2m- dimensional cotangent bundle of an m-dimensional configuration manifold $\boldsymbol{\mathcal{M}}$.

Let $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$be the momentum space and $\mathbf{Z}=\left(\mathbf{z^i},\mathbf{x^i}=\boldsymbol{\eta}^{\mathbf{i}^{(1)}},\mathbf{y^i}=\boldsymbol{\eta}^{\mathbf{i}^{(2)}}\right)=(\mathbf{z^i},\mathbf{X^i},\mathbf{Y^i}),\ \mathbf{1\le i\le m}$ its complex coordinates

Let almost complex structure $\mathbf{J}^*$ and Liouville form $\boldsymbol{\lambda}$ give by

$$\boldsymbol{\omega}=\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{z^i d\bar{z}_i+\bar{z}_i dz^i+x^i d\bar{x}_i+\bar{x}_i dx^i+y^i d\bar{y}_i+\bar{y}_i dy^i})\qquad(7)$$

$$\boldsymbol{\lambda}=(\mathbf{J}^*\boldsymbol{\omega})=\frac{\mathbf{1}}{\mathbf{2}}\mathbf{J}^*(\mathbf{z^i d\bar{z}_i+\bar{z}_i dz^i+x^i d\bar{x}_i+\bar{x}_i dx^i+y^i d\bar{y}_i+\bar{y}_i dy^i})$$

or

$$\boldsymbol{\lambda}=(\mathbf{J}^*\boldsymbol{\omega})=\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{z^i J^*(d\bar{z}_i)+\bar{z}_i J^*(dz^i)+x^i J^*(d\bar{x}_i)+\bar{x}_i J^*(dx^i)+y^i J^*(d\bar{y}_i)+\bar{y}_i J^*(dy^i)})$$

$$\boldsymbol{\lambda}=\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{-iz^i d\bar{z}_i+i\bar{z}_i dz^i-ix^i d\bar{x}_i+i\bar{x}_i dx^i-iy^i d\bar{y}_i+i\bar{y}_i dy^i})$$

or

$$\boldsymbol{\lambda}=\frac{\mathbf{1}}{\mathbf{2}}\mathbf{i}(\mathbf{-z^i d\bar{z}_i+\bar{z}_i dz^i-x^i d\bar{x}_i+\bar{x}_i dx^i-y^i d\bar{y}_i+\bar{y}_i dy^i}))\qquad(8)$$

such that $\boldsymbol{\omega}$ complex 1-form on $\mathbf{T_c^*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$.

If $\boldsymbol{\phi} = -\mathbf{d}\boldsymbol{\lambda}$ is closed Kahlerian form, then $\boldsymbol{\phi}$ is also a symplectic structure on $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$.

$$\boldsymbol{\phi} = -\mathbf{d}\boldsymbol{\lambda} = -\mathbf{d}\left(\frac{1}{2}\mathbf{i}\left(-\mathbf{z}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \bar{\mathbf{z}}_{\mathbf{i}}\mathbf{d}\mathbf{z}^{\mathbf{i}} - \mathbf{x}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \bar{\mathbf{x}}_{\mathbf{i}}\mathbf{d}\mathbf{x}^{\mathbf{i}} - \mathbf{y}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}} + \bar{\mathbf{y}}_{\mathbf{i}}\mathbf{d}\mathbf{y}^{\mathbf{i}}\right)\right)$$

$$\boldsymbol{\phi} = -\mathbf{d}\boldsymbol{\lambda} = -\mathbf{id}\left(-\mathbf{z}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \bar{\mathbf{z}}_{\mathbf{i}}\mathbf{d}\mathbf{z}^{\mathbf{i}}\right) - \mathbf{id}\left(-\mathbf{x}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \bar{\mathbf{x}}_{\mathbf{i}}\mathbf{d}\mathbf{x}^{\mathbf{i}}\right) - \mathbf{id}\left(-\mathbf{y}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}} + \bar{\mathbf{y}}_{\mathbf{i}}\mathbf{d}\mathbf{y}^{\mathbf{i}}\right)$$

$$\boldsymbol{\phi} = -\mathbf{d}\boldsymbol{\lambda} = -\mathbf{i}\left(\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{z}^{\mathbf{i}}\right) - \mathbf{i}\left(\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{x}^{\mathbf{i}}\right) - \mathbf{i}\left(\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{y}^{\mathbf{i}}\right) \qquad (9)$$

Let $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(2,0)}\boldsymbol{\mathcal{M}}\right)$ be momentum space with closed Kaehlerian form $\boldsymbol{\phi}$ .Consider that Hamiltonian vector field $\mathbf{Z}_{\mathbf{H}}$ associated Hamiltonian energy $\mathbf{H}$ is given by

$$\mathbf{Z} = \mathbf{Z}_{\mathbf{H}} = \mathbf{Z}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{z}^{\mathbf{i}}} + \bar{\mathbf{Z}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{z}}_{\mathbf{i}}} + \mathbf{X}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{x}^{\mathbf{i}}} + \bar{\mathbf{X}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{x}}_{\mathbf{i}}} + \mathbf{Y}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{y}^{\mathbf{i}}} + \bar{\mathbf{Y}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{y}}_{\mathbf{i}}} \qquad \mathbf{1} \le \mathbf{i} \le \mathbf{m}$$

From the isomorphism given in, we calculate by

$$\begin{aligned}\mathbf{i}_{\mathbf{Z}_{\mathbf{H}}}\boldsymbol{\phi} &= \mathbf{i}_{\mathbf{Z}_{\mathbf{H}}}(-\mathbf{d}\boldsymbol{\lambda})\\ &= \left(\mathbf{Z}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{z}^{\mathbf{i}}} + \bar{\mathbf{Z}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{z}}_{\mathbf{i}}} + \mathbf{X}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{x}^{\mathbf{i}}} + \bar{\mathbf{X}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{x}}_{\mathbf{i}}} + \mathbf{Y}^{\mathbf{i}}\frac{\partial}{\partial\mathbf{y}^{\mathbf{i}}} + \bar{\mathbf{Y}}_{\mathbf{i}}\frac{\partial}{\partial\bar{\mathbf{y}}_{\mathbf{i}}}\right)\Big(-\mathbf{i}\left(\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{z}^{\mathbf{i}}\right) - \mathbf{i}\left(\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{x}^{\mathbf{i}}\right)\\ &\quad - \mathbf{i}\left(\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}}\wedge\mathbf{d}\mathbf{y}^{\mathbf{i}}\right)\Big)\end{aligned}$$

$$\mathbf{i}_{\mathbf{Z}_{\mathbf{H}}}\boldsymbol{\phi} = \mathbf{i}\bar{\mathbf{Z}}_{\mathbf{i}}\mathbf{d}\mathbf{z}^{\mathbf{i}} + \mathbf{i}\mathbf{Z}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \mathbf{i}\bar{\mathbf{X}}_{\mathbf{i}}\mathbf{d}\mathbf{x}^{\mathbf{i}} + \mathbf{i}\mathbf{X}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \mathbf{i}\bar{\mathbf{Y}}_{\mathbf{i}}\mathbf{d}\mathbf{y}^{\mathbf{i}} + \mathbf{i}\mathbf{Y}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}} \qquad (\mathbf{10})$$

On the other hand, we obtain as

$$\mathbf{dH} = \frac{\partial\mathbf{H}}{\partial\mathbf{z}^{\mathbf{i}}}\mathbf{d}\mathbf{z}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{z}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\mathbf{x}^{\mathbf{i}}}\mathbf{d}\mathbf{x}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{x}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\mathbf{y}^{\mathbf{i}}}\mathbf{d}\mathbf{y}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{y}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}} \qquad (\mathbf{11})$$

the differential of Hamiltonian energy. From $\mathbf{i}_{\mathbf{Z}_{\mathbf{H}}}\boldsymbol{\phi} = \mathbf{dH}$, we find as

$$\begin{aligned}\mathbf{i}_{\mathbf{Z}_{\mathbf{H}}}\boldsymbol{\phi} = \mathbf{dH} &= \bar{\mathbf{Z}}_{\mathbf{i}}\mathbf{d}\mathbf{z}^{\mathbf{i}} + \mathbf{i}\mathbf{Z}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \mathbf{i}\bar{\mathbf{X}}_{\mathbf{i}}\mathbf{d}\mathbf{x}^{\mathbf{i}} + \mathbf{i}\mathbf{X}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \mathbf{i}\bar{\mathbf{Y}}_{\mathbf{i}}\mathbf{d}\mathbf{y}^{\mathbf{i}} + \mathbf{i}\mathbf{Y}^{\mathbf{i}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}}\\ &= \frac{\partial\mathbf{H}}{\partial\mathbf{z}^{\mathbf{i}}}\mathbf{d}\mathbf{z}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{z}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{z}}_{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\mathbf{x}^{\mathbf{i}}}\mathbf{d}\mathbf{x}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{x}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{x}}_{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\mathbf{y}^{\mathbf{i}}}\mathbf{d}\mathbf{y}^{\mathbf{i}} + \frac{\partial\mathbf{H}}{\partial\bar{\mathbf{y}}_{\mathbf{i}}}\mathbf{d}\bar{\mathbf{y}}_{\mathbf{i}} \qquad (\mathbf{12})\end{aligned}$$

Or

$$\mathbf{Z}_{\mathbf{H}} = \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\bar{\mathbf{z}}_{\mathbf{i}}}\frac{\partial}{\partial\mathbf{z}^{\mathbf{i}}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\mathbf{z}^{\mathbf{i}}}\frac{\partial}{\partial\bar{\mathbf{z}}_{\mathbf{i}}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\bar{\mathbf{x}}_{\mathbf{i}}}\frac{\partial}{\partial\mathbf{x}^{\mathbf{i}}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\mathbf{x}^{\mathbf{i}}}\frac{\partial}{\partial\bar{\mathbf{x}}_{\mathbf{i}}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\bar{\mathbf{y}}_{\mathbf{i}}}\frac{\partial}{\partial\mathbf{y}^{\mathbf{i}}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\partial\mathbf{H}}{\partial\mathbf{y}^{\mathbf{i}}}\frac{\partial}{\partial\bar{\mathbf{y}}_{\mathbf{i}}} \qquad (\mathbf{13})$$

$$\mathbf{1} \le \mathbf{i} \le \mathbf{m}$$

Let $\{\mathbf{Z} = (\mathbf{z}^{\mathbf{i}}, \bar{\mathbf{z}}_{\mathbf{i}}, \mathbf{x}^{\mathbf{i}}, \bar{\mathbf{x}}_{\mathbf{i}}, \mathbf{y}^{\mathbf{i}}, \bar{\mathbf{y}}_{\mathbf{i}}) : \mathbf{1} \le \mathbf{i} \le \mathbf{m}\}$ be the complex coordinates in the momentum space. Suppose that the curve

$$\boldsymbol{\alpha}: \mathbf{I} \subset \mathbf{C} \rightarrow \mathbf{T}\boldsymbol{\mathcal{M}}$$

be an integral curve of Hamiltonian vector field $\mathbf{Z_H}$, i.e.,

$$\mathbf{Z_H}\big(\boldsymbol{\alpha}(\mathbf{t})\big) = \dot{\boldsymbol{\alpha}} \quad , \mathbf{t} \in \mathbf{I}.$$

In the local coordinates we have

$$\boldsymbol{\alpha}(\mathbf{t}) = \Big(\mathbf{z^i}(\mathbf{t}), \bar{\mathbf{z}}_\mathbf{i}(\mathbf{t}), \mathbf{x^i}(\mathbf{t}), \bar{\mathbf{x}}_\mathbf{i}(\mathbf{t}), \mathbf{y^i}(\mathbf{t}), \bar{\mathbf{y}}_\mathbf{i}(\mathbf{t})\Big),$$

And

$$\dot{\boldsymbol{\alpha}}(\mathbf{t}) = \frac{\mathbf{dz^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} + \frac{\mathbf{d}\bar{\mathbf{z}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}} + \frac{\mathbf{dx^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} + \frac{\mathbf{d}\bar{\mathbf{x}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}} + \frac{\mathbf{dy^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} + \frac{\mathbf{d}\bar{\mathbf{y}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}} \qquad (\mathbf{14})$$

the Hamiltonian vector field on momentum space $\mathbf{T^*_c}\big(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\big)$ with closed Kaehlerian form $\boldsymbol{\phi}$.Now, from $\mathbf{Z_H}\big(\boldsymbol{\alpha}(\mathbf{t})\big) = \dot{\boldsymbol{\alpha}}$,

$$\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{z^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{x^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}} + \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} - \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{y^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}}$$
$$= \frac{\mathbf{dz^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} + \frac{\mathbf{d}\bar{\mathbf{z}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}} + \frac{\mathbf{dx^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} + \frac{\mathbf{d}\bar{\mathbf{x}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}} + \frac{\mathbf{dy^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} + \frac{\mathbf{d}\bar{\mathbf{y}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}}$$

then we infer the following equations

$$\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} = \frac{\mathbf{dz^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{z^i}} \rightarrow \frac{\mathbf{dz^i}}{\mathbf{dt}} = \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}}$$

$$-\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{z^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}} = \frac{\mathbf{d}\bar{\mathbf{z}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{z}}_\mathbf{i}} \rightarrow \frac{\mathbf{d}\bar{\mathbf{z}}_\mathbf{i}}{\mathbf{dt}} = -\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{z^i}}$$

$$\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} = \frac{\mathbf{dx^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{x^i}} \rightarrow \frac{\mathbf{dx^i}}{\mathbf{dt}} = \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}}$$

$$-\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{x^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}} = \frac{\mathbf{d}\bar{\mathbf{x}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{x}}_\mathbf{i}} \rightarrow \frac{\mathbf{d}\bar{\mathbf{x}}_\mathbf{i}}{\mathbf{dt}} = -\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{x^i}}$$

$$\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} = \frac{\mathbf{dy^i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\mathbf{y^i}} \rightarrow \frac{\mathbf{dy^i}}{\mathbf{dt}} = \frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}}$$

$$-\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{y^i}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}} = \frac{\mathbf{d}\bar{\mathbf{y}}_\mathbf{i}}{\mathbf{dt}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\bar{\mathbf{y}}_\mathbf{i}} \rightarrow \frac{\mathbf{d}\bar{\mathbf{y}}_\mathbf{i}}{\mathbf{dt}} = -\frac{\mathbf{1}}{\mathbf{i}}\frac{\boldsymbol{\partial}\mathbf{H}}{\boldsymbol{\partial}\mathbf{y^i}}$$

which are called complex Hamiltonian equations on momentum space $\mathbf{T^*_c}\big(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\big)$. we have the complex

Hamiltonian equations given by

$$\frac{d\bar{z}^i}{dt}=\frac{1}{i}\frac{\partial H}{\partial z_i} \quad , \qquad \frac{dz_i}{dt}=-\frac{1}{i}\frac{\partial H}{\partial \bar{z}^i}$$

$$\frac{dx^i}{dt}=\frac{1}{i}\frac{\partial H}{\partial \bar{x}_i} \quad , \qquad \frac{d\bar{x}_i}{dt}=-\frac{1}{i}\frac{\partial H}{\partial x^i}$$

$$\frac{dy^i}{dt}=\frac{1}{i}\frac{\partial H}{\partial \bar{y}_i} \quad , \qquad \frac{d\bar{y}_i}{dt}=-\frac{1}{i}\frac{\partial H}{\partial y^i} \tag{15}$$

Thus, by complex Hamiltonian equations ,we may call the equations obtained in (15) on $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\right)$. Then the quartet $(\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\right), \boldsymbol{\phi}_{\mathbf{H}}, \mathbf{Z}_{\mathbf{H}})$ is named mechanical system with

4. Conclusions

The solutions of the Hamiltonian equations determined by (15) on the mechanical system $(\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\right), \boldsymbol{\phi}_{\mathbf{L}}, \mathbf{Z}_{\mathbf{H}})$are the paths of vector field $\mathbf{Z}_{\mathbf{H}}$ on $\mathbf{T}_{\mathbf{c}}^{*}\left(\mathbf{J}^{(\mathbf{2,0})}\boldsymbol{\mathcal{M}}\right)$.

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